The Algebra of Mohammed Ben Musa

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MENSURATION.

KNOW that the meaning of the expression "one by one" is mensuration : one yard (in length) by one yard (in breadth) being understood

Every quadrangle of equal sides and angles, which has one yard for every side, has also one for its area. Has such a quadrangle two yards for its side, then the area of the quadrangle is four times the area of a quadrangle, the side of which is one yard. The same takes place with three by three, and so on, ascending or descending: for instance, a half by a half, which gives a quarter, or other fractions, always following the same rule. A quadrate, every side of which is half a yard, is (51) equal to one-fourth of the figure which has one yard for its side. In the same manner, one-third by one-third, or one-fourth by one-fourth, or one-fifth by one-fifth, or two-thirds by a half, or more or less than this, always according to the same rule.

One side of an equilateral quadrangular figure, taken once, is its root; or if the same be multiplied by two, then it is like two of its roots, whether it be small or great.

If you multiply the height of any equilateral triangle by the moiety of the basis upon which the line marking the height stands perpendicularly, the product gives the area of that triangle.

In every equilateral quadrangle, the product of one diameter multiplied by the moiety of the other will be equal to the area of it.

In any circle, the product of its diameter, multiplied by three and one-seventh, will be equal to the periphery. This is the rule generally followed in practical life, though it is not quite exact. The geometricians have two other methods. One of them is, that you multiply the diameter by itself; then by ten, and hereafter take the root of the product; the root will be the periphery. The other method is used by the astronomers among them: it is this, that you multiply the diameter by sixty-two thousand eight hundred and thirty-two and then divide the product by twenty thousand; the quotient is the periphery. Both methods come very nearly to the same effect. [1]

If you divide the periphery by three and one-seventh, the quotient is the diameter.

The area of any circle will be found by multiplying the moiety of the circumference by the moiety of the diameter; since, in every polygon of equal sides and angles, such as triangles, quadrangles, pentagons, and so on, the area is found by multiplying the moiety of the circumference by the moiety of the diameter of the middle circle that may be drawn through it.

If you multiply the diameter of any circle by itself, and subtract from the product one-seventh and half one-seventh of the same, then the remainder is equal to the area of the circle. This comes very nearly to the same result with the method given above, [2]

Every part of a circle may be compared to a bow. It must be either exactly equal to half the circumference, or less or greater than it. This may be ascertained by the arrow of the bow. When this becomes equal to the moiety of the chord, then the arc is exactly the moiety of the circumference: is it shorter than the moiety of the chord, then the bow is less than half the circumference; is the arrow longer than half the chord, then the bow comprises more than half the circumference.

If you want to ascertain the circle to which it belongs, multiply the moiety of the chord by itself, divide it by the arrow, and add the quotient to the arrow, the sum is the diameter of the circle to which this bow belongs.

If you want to compute the area of the bow, multiply the moiety of the diameter of the circle by the moiety of the bow, and keep the product in mind. Then subtract the arrow of the bow from the moiety of the diameter of the circle, if the bow is smaller than half the circle; or if it is greater than half the circle, subtract half the diameter of the circle from the arrow of the bow. Multiply the remainder by the moiety of the chord of the bow, and subtract the product from that which you have kept in mind if the bow is smaller than the moiety of the circle, or add it thereto if the bow is greater than half the circle. The sum after the addition, or the remainder after the subtraction, is the area of the bow.

The bulk of a quadrangular body will be found by multiplying the length by the breadth, and then by the height.

If it is of another shape than the quadrangular (for instance, circular or triangular), so, however, that a line representing its height may stand perpendicularly

[1] The three formulas are,
1st, $3\frac{1}{7}d = p$ i.e. $3.1428d$
2d, $\sqrt{10d^2} = p$ i.e. $.16227d$
3d, $\frac{d \times 62832}{20000} = p$ i.e.$3.1416d$

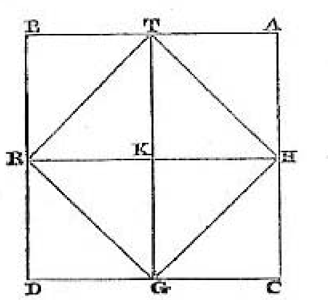
[2] The area of a circle whosse diameter is d is $\pi\frac{d^2}{4} = \frac{22}{7\times 4}d^2 = (1 - \frac{1}{7} - \frac{1}{2\times 7})d^2$.

on its basis, and yet be parallel to the sides, you must calculate it by ascertaining at first the area of its basis. This, multiplied by the height, gives the bulk of the body.

Cones and pyramids, such as triangular or quadrangular ones, are computed by multiplying one-third of the area of the basis by the height.

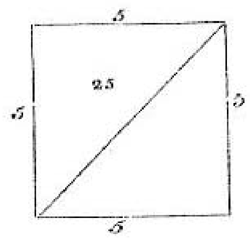
Observe, that in every rectangular triangle the two short sides, each multiplied by itself and the products added together, equal the product of the long side multiplied by itself.

The proof of this is the following. We draw a quadrangle, with equal sides and angles $ABCD$. We divide the line AC into two moieties in the point H, from which we draw a parallel to the point R. Then we divide, also, the line AB into two moieties at the point T, and draw a parallel to the point G. Then the quadrate $ABCD$ is divided into four quadrangles of equal sides and angles, and of equal area; namely., the squares AK, CK, BK, and DK. Now, we draw from the point H to the point T a line which divides the quadrangle AK into two equal parts: thus there arise two triangles from the quadrangle, namely, the triangles ATH and HKT. We know that AT is the moiety of AB, and that AH is equal to it, being the moiety of AC; and the line TH joins them opposite the right angle. In the same manner we draw lines from T to R, and from R to G, and from G to H. Thus from all the squares eight equal triangles arise, four of which must, consequently, be equal to the moiety of the great quadrate AD. We know that the line AT multiplied by itself is like the area of two triangles, and AK gives the area of two triangles equal to them; the sum of them is therefore four triangles. But the line HT, multiplied by itself, gives likewise the area of four such triangles. We perceive, therefore, that the sum of AT multiplied by itself, added to AH multiplied by itself, is equal to TH multiplied by itself. This is the observation which we were desirous to elucidate. Here is the figure to it:

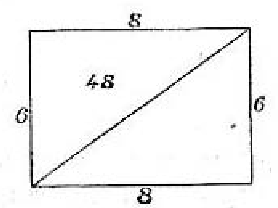


Quadrangles are of five kinds: firstly, with right angles and equal sides; secondly, with right angles and unequal sides; thirdly, the rhombus, with equal sides and unequal angles; fourthly, the rhomboid, the length of which differs from its breadth, and the angles of which are unequal, only that the two long and the two short sides are respectively of equal length; fifthly, quadrangles with unequal sides and angles.

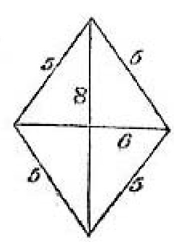
First kind. – The area of any quadrangle with equal sides and right angles, or with unequal sides and right angles, may be. found by multiplying the length by the breadth. The product is the area. For instance: a quadrangular piece of ground, every side of which has five yards, has an area of five-and-twenty square yards. Here is its figure



Second kind. – A quadrangular piece of ground, the two long sides of which are of eight yards each, while the breadth is six.. You find the area by multiplying six by eight, which yields forty-eight yards. Here is the figure to it:

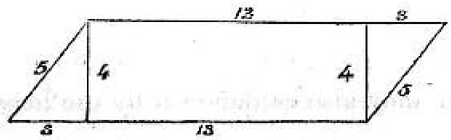


Third kind, the Rhombus. – Its sides are equal: let each of them be five, and let its diagonals be, the one eight and the other six yards. You may then compute the area, either from one of the diagonals, or from both. As you know them both, you multiply the one by the moiety of the other, the product is the area: that is to say, you multiply eight by three, or six by four; this yields twenty-four yards, which is the area. If you know only one of the diagonals, then you are aware, that there are two triangles, two sides of each of which have every one five yards, while the third is the diagonal. Hereafter you can make the computation according to the rules for the triangles. [3] This is the figure:



The fourth kind, or Rhomboid, is computed in the same way as the rhombus. Here is the figure to it:

[3]If the two diagonals are d and d', and the side s, the area of the rhombus is $\frac{dd'}{2} = d \times \sqrt{s^2 - \frac{d^2}{4}}$.

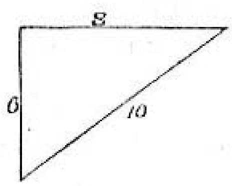


The other quadrangles are calculated by drawing a diagonal, and computing them as triangles.

Triangles are of three kinds, acute-angular, obtuse-angular, or rectangular. The peculiarity of the rectangular triangle is, that if you multiply each of its two short sides by itself, and then add them together, their sum will be equal to the long side multiplied by itself. The character of the acute-angled triangle is this : if you multiply every one of its two short sides by itself, and add the products, their sum is more than the long side alone multiplied by itself. The definition of the obtuse-angled triangle is this : if you multiply its two short sides each by itself, and then add the products, their sum is less than the product of the long side multiplied by itself.

The rectangular triangle has two cathetes and an hypotenuse. It may be considered as the moiety of a quadrangle. You find its area by multiplying one of its cathetes by the moiety of the other. The product is the area.

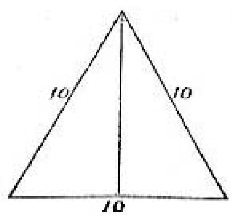
Examples. – A rectangular triangle; one cathete being (58) six yards, the other eight, and the hypotenuse ten. You make the computation by multiplying six by four: this gives twenty-four, which is the area. Or if you prefer, you may also calculate it by the height, which rises perpendicularly from the longest side of it: for the two short sides may themselves be considered as two heights. If you prefer this, you multiply the height by the moiety of the basis. The product is the area. This is the figure:



Second kind. – An equilateral triangle with acute angles, every side of which is ten yards long. Its area may be ascertained by the line representing its height and the point from which it rises. [4] Observe, that in every isosceles triangle, a line to represent the height drawn to the basis rises from the latter in a right angle, and the point from which it proceeds is always situated in the midst of the basis; if, on the contrary, the two sides are not equal, then this point never lies in the middle of the basis. In the case now before us we perceive, that towards whatever side we may draw the line which is to represent the height, it must necessarily always fall in the middle of it, where the length of the basis is five. Now the height will be

[4] The height of the equilateral triangle whose side is 10, is $\sqrt{10^2 - 5^2} = \sqrt{75}$ and the area of the triangle is $5\sqrt{75} = 25\sqrt{3}$

ascertained thus. You multiply five by itself; then multiply one of the sides, that is ten, by itself, which gives a hundred. Now you subtract from this the product of five multiplied by itself, which is twenty-five. The remainder is seventy-five, the root of which is the height. This is a line common to two rectangular triangles. If you want to find the area, multiply the root of seventy-five by the moiety of the basis, which is five. This you perform by multiplying at first five by itself; then you may say, that the root of seventy-five is to be multiplied by the root of twenty-five. Multiply seventy-five by twenty-five. The product is one thousand eight hundred and seventy-five ; take its root, it is the area: it is forty-three and a little. [5] Here is the figure:

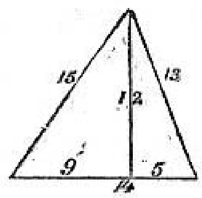


There are also acute-angled triangles, with different sides. Their area will be found by means of the line representing the height and the point from which it proceeds. Take, for instance, a triangle, one side of which is fifteen yards, another fourteen, and the third thirteen yards. In order to find the point from which the line marking the height does arise, you may take for the basis any side you choose; e. g. that which is fourteen yards long. The point from which the line representing the height does arise, lies in this basis at an unknown distance from either of the two other sides. Let us try to find its unknown distance from the side which is thirteen yards long. Multiply this distance by itself; it becomes an [unknown] square. Subtract this from thirteen multiplied by itself; that is, one hundred and sixty-nine. The remainder is one hundred and sixty-nine less a square. The root from this is the height. The remainder of the basis is fourteen less thing. We multiply this by itself; it becomes one hundred and ninety-six, and a square less twenty-eight things. We subtract this from fifteen multiplied by itself; the remainder is twenty-nine dirhems and twenty-eight things less one square. The root of this is the height. As, therefore, the root of this is the height, and the root of one hundred and sixty-nine less square is the height likewise, we know that, they both are the same. [6] Reduce them, by removing square against square, since both are negatives. There remain twenty-nine [dirhems] plus twenty-eight things, which are equal to one hundred and sixty-nine. Subtract now twenty-nine from one hundred and sixty-nine. The remainder is one hundred and forty, equal

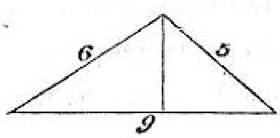
[5] The root is 43.3 +

[6] $\sqrt{169} - x^2 = 29 + 28x - x^2$
$163 = 20 + 28x$
$140 = 28x$
$5 = x$

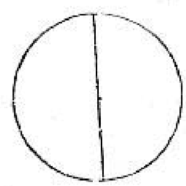
to twenty-eight things. One thing is, consequently, five. This is the distance of the said point from the side of thirteen yards. The complement of the basis towards the other side is nine. Now in order to find the height, you multiply five by itself, and subtract it from the contiguous side, which is thirteen, multiplied by itself. The remainder is one hundred and forty-four. Its root is the height. It is twelve. The height forms always two right angles with the basis, and it is called the *column*, on account of its standing perpendicularly. Multiply the height into half the basis, which is seven. The product is eighty-four, which is the area. Here is the figure :



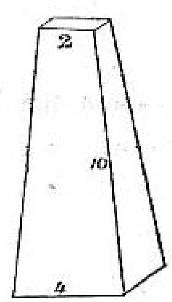
The third species is that of the obtuse-angled triangle with one obtuse angle and sides of different length. For instance, one side being six, another five, and the third nine. The area of such a triangle will be found by means of the height and of the point from which a line representing the same arises. This point can, within such a triangle, lie only in its longest side. Take therefore this as the basis : for if you choose to take one of the short sides as the basis, then this point would fall beyond the triangle. You may find the distance of this point, and the height, in the same manner, which I have shown in the acute-angled triangle; the whole computation is the same. Here is the, figure;



We have above treated at length of the circles, of their qualities and their computation. The following is an example: If a circle has seven for its diameter, then it has twenty-two for its circumference. Its area you find in the following manner: Multiply the moiety of the diameter, which Is three and a half, by the moiety of the circumference, which Is eleven. The product is thirty-eight and a half, which is the area. Or you may also multiply the diameter, which is seven, by itself: this is forty-nine; subtracting herefrom one-seventh and half one-seventh, which is ten and a half, there remain thirty-eight and a half, which is the area. Here is the figure:



If some one inquires about the bulk of a pyramidal pillar, its base being four yards by four yards, its height ten yards, and the dimensions at its upper extremity two yards by two yards; then we know already that every pyramid is decreasing towards its top, and that one-third of the area of its basis, multiplied by the height, gives its bulk. The present pyramid has no top. We must therefore seek to ascertain what is wanting in its height to complete the top. We observe, that the proportion of the entire height to the ten, which we have now before us, is equal to the proportion of four to two. Now as two is the moiety of four, ten must likewise be the moiety of the entire height, and the whole height of the pillar must be twenty yards; At present we take one-third of the area of the basis, that is, five and one-third, and multiply it by the lengthj which is twenty. The product is one hundred and six yards and two-thirds. Herefrom we must then subtract the piece, which we have added in order to complete the pyramid. This we perform by multiplying one and one-third, which is one-third of the product of two by two, by ten : this gives thirteen and a third. This is the piece which we have added in order to complete the pyramid. Subtracting this from one hundred and six yards and two-thirds, there remain ninety-three yards and one-third : and this is the bulk of the mutilated pyramid. This is the figure:



If the pillar has a circular basis, subtract one-seventh and half a seventh from the product of the diameter multiplied by itself, the remainder is the basis. If some one says: "There is a triangular piece of land, two of its sides having ten yards each, and the basis twelve; what must be the length of one side of a quadrate situated within such a triangle?" the solution is this. At first you ascertain the height of the triangle, by multiplying the moiety of the basis, (which is six) by itself, and subtracting the product, which is thirty-six, from one of the two short sides multiplied itself, which is one hundred; the remainder is sixty-four: take the root from this; It is eight. This is the height of the triangle. Its area is, therefore, forty-eight yards: such being the product of the height multiplied by the moiety of the basis, which is six. Now we assume that one side of the quadrate inquired for is thing. We multiply it by itself; thus it becomes a square, which we keep in mind. We know that there must remain two triangles on the two sides of the quadrate, and one above it. The two triangles on both sides of it are equal to each other : both having the same height and being rectangular. You find their area by multiplying thing by six less half a thing, which gives six things less half a square. This is the area of both the triangles on the two sides of the quadrate together. The area of the upper triangle will be found by multiplying eight less thing, which is the height, by half one thing. The product is four things less half a square. This altogether is equal to the area of the quadrate plus that of the three triangles: or, ten things are equal to forty-eight, which is the area of the great triangle. One thing from this is four yards and four-fifths of a yard; and this is the length of any side of the quadrate. Here is the figure: